**Problem description and mathematical formulation**

The multi-depot multi-trip vehicle routing problem with time windows (MD-MT-VRPTW) that the dissertation examines can be defined as follows. Let be the network graph of the problem, where is the set of the vertices which includes a set of depot location and a set of customer location , and is the set of arcs between two vertices in set V. Every arc in set E has travel cost and travel time . is the set of all the vehicles, which are distributed at the depot locations they belong to. For each vehicle , multiple trips can be made within daily working hours and the maximum volume of oil to be collected per vehicle is . Each trip for each vehicle will be coded and recorded in set , representing the trips made by each vehicle. For each customer , each has their own daily opening time and closing time . Also, each customer has their order quantity for oil and a vehicle will spend service time to complete the oil recovery after arriving at the customer's location. When the vehicle returns to the depot, additional time is required to store the recovered oil from the vehicle at the depot location. When the customer submits an order, a desired lead time , preferred date and closing date are given. During a vehicle's trip, the vehicle starts from the depot and ends at the depot. However, if the time that vehicle returns to the depot after servicing a customer exceeds the rest of the day's working hours, the vehicle will end the trip at that customer's location for an overnight stay and start the trip at the customer's location on the next day. The vehicle will incur an overnight cost for each overnight stay and the overnight situation will only occur on the vehicle's last trip of the day.

The aim of MD-MT-VRPTW is to schedule the daily trips of vehicles within the maximum lead time of all customers and to minimize the total travel and overnight costs. In addition, the following conditions need to be satisfied:

(1) Each customer should only be served once

(2) Two consecutive trips on the same vehicle cannot overlap

(3) Vehicle needs to arrive at customer 's location after the opening time and before the closing time

(4) The total amount of oil recovered from the customer between the time the vehicle leaves the depot and the next time it returns to the depot cannot exceed the total capacity of the vehicle .

(5) Vehicle needs to complete the customer's order within the defined number of working days .

(6) Vehicle needs to finish the customer’s order on the preferred date and to avoid servicing the customer on the closing date .

The notations and mathematical formulation of the MD-MT-VRPTW are introduced as follows:

**Sets and Indices**

|  |  |
| --- | --- |
|  | Set of depots |
|  | Set of customers |
|  | Set of vehicles |
|  | Set of trips for each vehicle |
|  | Date that vehicle work |
|  | Index of Vertices belong to M∪N |
|  | Index of a Vehicle |
|  | Index of a Trip |
|  | Index of date |

**Parameters**

|  |  |
| --- | --- |
|  | Opening time at customer *i* |
|  | Closing time at customer *i* |
|  | Lead time of customer *i* |
|  | Preferred date of customer *i* |
|  | Closed date of customer *i* |
|  | Travel cost between Vertices *i* and Vertices *j* |
|  | Travel time between Vertices *i* and Vertices *j* |
|  | Order quantity of customer *i* |
|  | Capacity of Vehicle |
|  | Service time of customer *i* |
|  | Service time of depot |
|  | Working time horizon of vehicle |
| O | Overnight cost |

**Decision Variables**

|  |  |
| --- | --- |
|  | Binary variable equal to 1 if vehicle visit vertices after visiting vertices in trip at date , otherwise equal to 0 |
|  | Binary variable equal to 1 if vehicle visit vertices in trip at date , otherwise equal to 0 |
|  | Binary variable equal to 1 if vehicle responsible for trip , otherwise equal to 0 |
|  | Binary variable equal to 1 if the start location of tripserve by vehicle is not depot location at date , otherwise equal to 0 |
|  | Binary variable equal to 1 if vehicle should overnight at the end of trip at date , otherwise equal to 0 |
|  | Binary variable equal to 1 if the first customer in trip by vehicle is customerat date , otherwise equal to 0 |
|  | Binary variable equal to 1 if the last customer in trip by vehicle is customer at date , otherwise equal to 0 |
|  | Binary variable equal to 1 if vehicle belongs to depot |
|  | Time point that vehicle arrive customer in trip at date |
|  | Time point that vehicle start trip at date |
|  | Time point that vehicle end trip *w* at date |
|  | Time point that vehicle start to work at date |
|  | Time point that vehicle end the work at date |
|  | Date that customeris served |

**Objective Function**

|  |  |
| --- | --- |
|  | (1) |

**Subject to**

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|  | (28) |
|  | (29) |
|  | (30) |

The objective function (1) minimize the total travel cost and the overnight cost of all the trips. Constraint (2) ensure that each customer is served only once. Constraint (3) guarantee that only one vehicle is responsible for each journey. Constraint (4) ensure that vehicle has performed trip before it will perform trip . Constraint (5) guarantee that trips which do not start and end at the customer's location start from the depot and return to the depot at the end of the trip. Constraint (6) ensure that total volume of oil recovered in trips which do not start and end at the customer's location should not exceed the vehicle capacity . Constraint (7) ensure that the volume of oil recovered in the vehicle does not exceed the vehicle capacity before vehicle returning to the depot. Constraint (8)(9) that the first customer of a trip and the last customer of a trip are included in the trip schedule. Constraint (10) explain the relationship between the destination of the overnight trip of vehicle and the start point of the first trip on the next day. Constraint (11)(12) ensure that vehicle k arrives at customer i within the operating hours of customer i. Constraint (13) address the time relationship between two consecutive trips of vehicle k in the same date. Constraint (14) address the time relationship between two consecutive customer i and j in trip w. Constraint (15)(16) illustrates the conditions to be met by vehicle k to overnight. Constraint (17)-(20) address the time relationship of trip starting time and trip ending time in different scenario. Constraint (21) address the time relationship between the starting time of trip and the starting time of the day in the same vehicle. Constraint (22) address the time relationship between the ending time of trip and the ending time of the day in the same vehicle. Constraint (23) is the daily working time constraint of a vehicle. Constraint (24)-26) is the date constraint of lead time, preferred date and closing date. Constraint (27)-(30) is the binary constraint of decision variables.